Quantum Error Correction

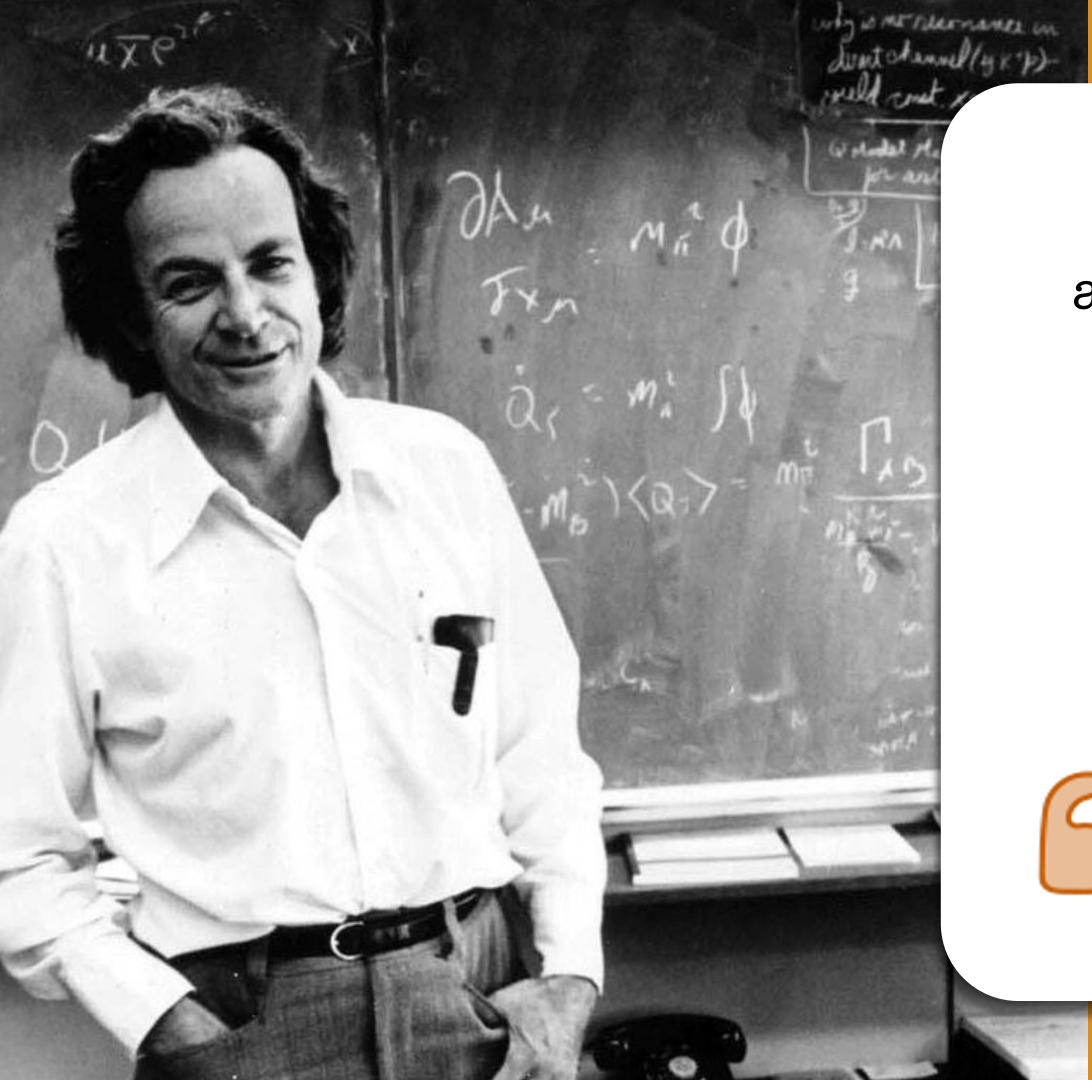
Aaron Jarmusch, Ph.D. Student, University of Delaware
CRPL
<jarmusch@udel.edu>

QSEG 851: Advanced Topics in Quantum Information

CRPL

University of Delaware

05/15/2025



Nature isn't classical... and if you want to make a simulation of nature, you'd better make it quantum mechanical

- Richard P. Feynman (1982)

Programming on a Computer

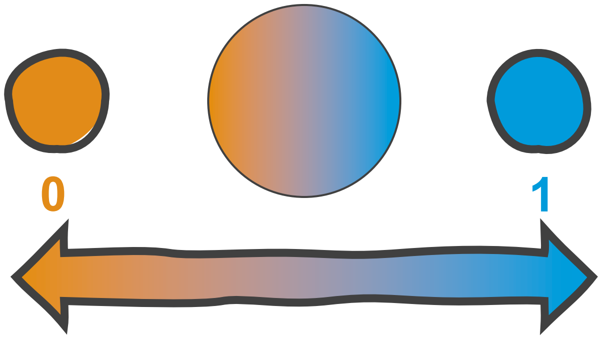
Superposition – A qubit can be 0 and 1 at the same time, until you measure.

Entanglement – Two qubits become linked.

Decoherence – Info. leaks into the env where information stored behave more classically.

Qubit

(Quantum Computing)



CRPL

University of Delaware

Classical Error Correction

3-bit repetition code - The idea is to repeat each bit multiple times.

Encoding

0 -> 000
1-> 111

Decoding

majority(a, b, c)

Corrects up to one **bit flip** on any of the three bits.

Scaling up the number of repeated bits can improve the error correction.

CRPL

UNIVERSITY OF DELAWARE

No-cloning theorem – quantum states cannot be perfectly copied

The **no-cloning theorem** prevents copying qubits for backup

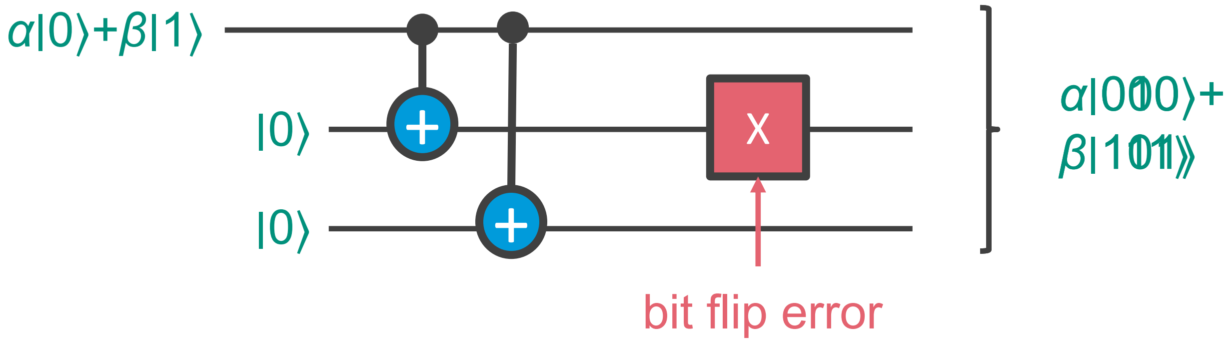
… so classical error correction **wouldn’t work.**

Qubit repetition codes

The **3-bit repetition code** can be used to encode a qubit.

Encoding

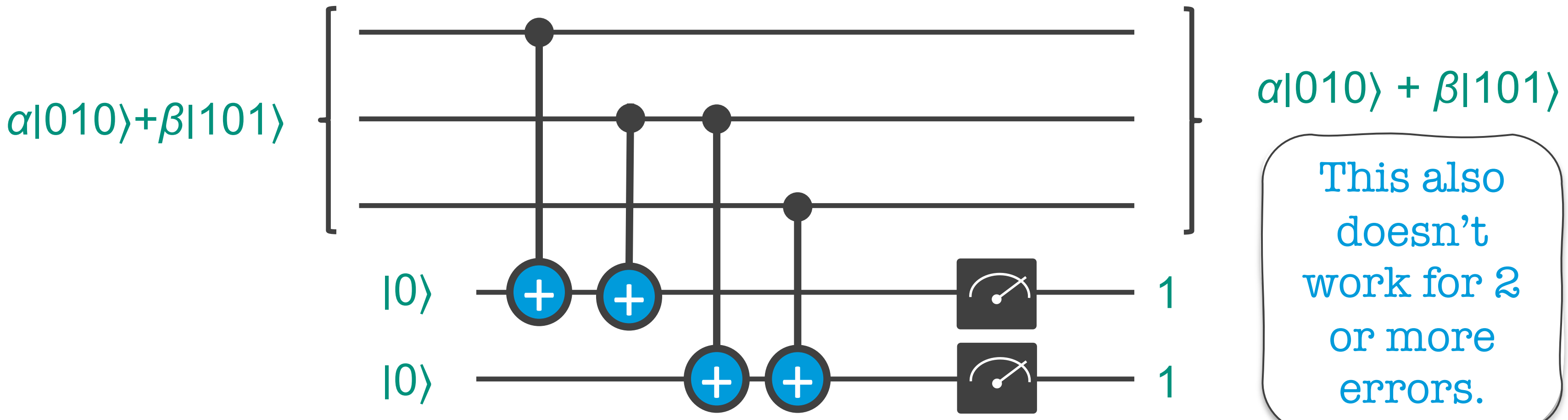
$\alpha|0\rangle+\beta|1\rangle \mapsto \alpha|000\rangle+\beta|111\rangle$



Option 1: Measure in

Option 2: Measure the parity of the standard basis states

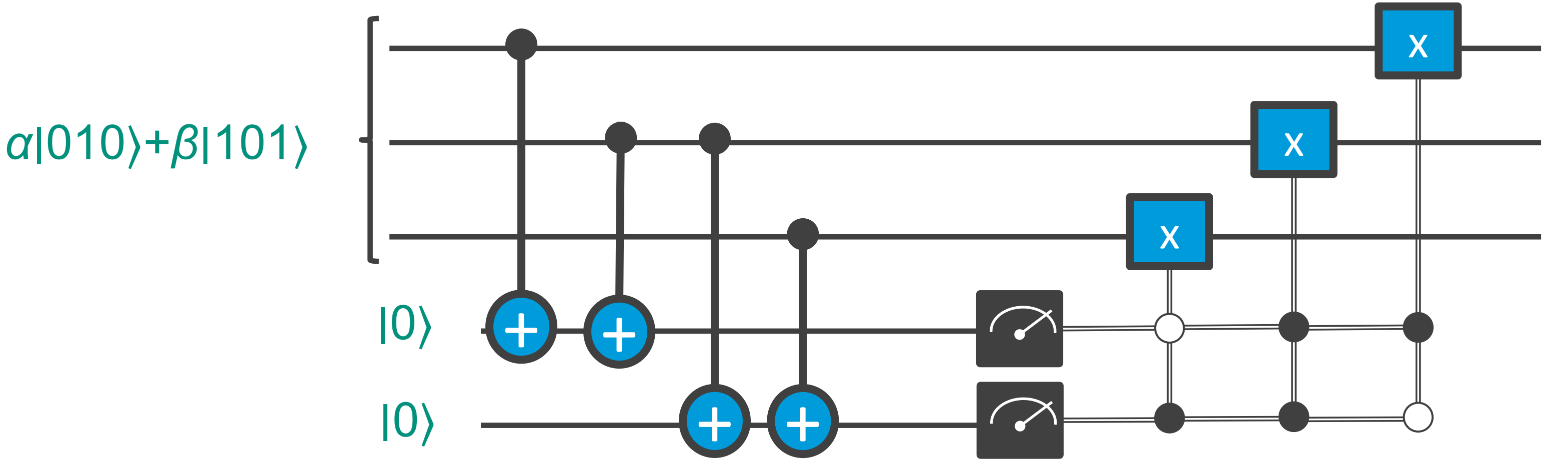
Qubit repetition codes – **bit flip error**



This also doesn't work for 2 or more errors.

This doesn't cause the state to collapse so we have a chance to correct the qubit.

Qubit repetition codes – **bit** flip error

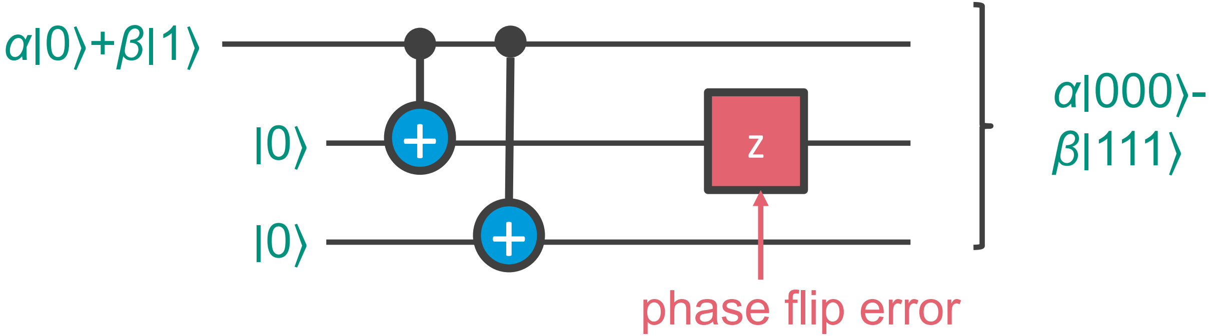


Qubit repetition codes – **phase** flip error

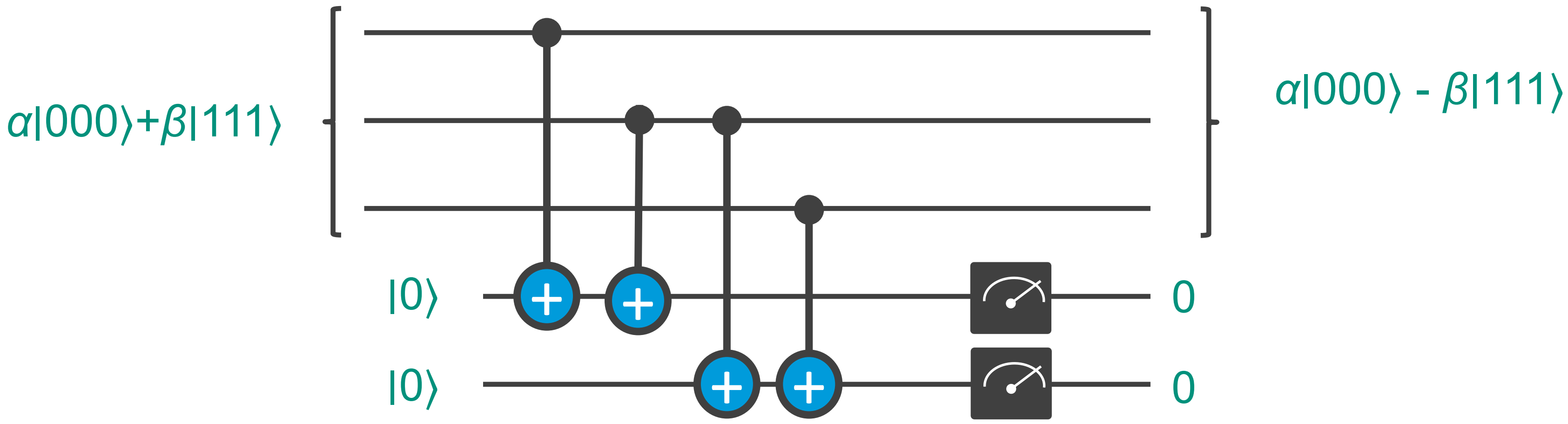
The **3-bit repetition code** can be used to encode a qubit.

Encoding

$\alpha|0\rangle+\beta|1\rangle \mapsto \alpha|000\rangle+\beta|111\rangle$



Qubit repetition codes – **phase** flip error



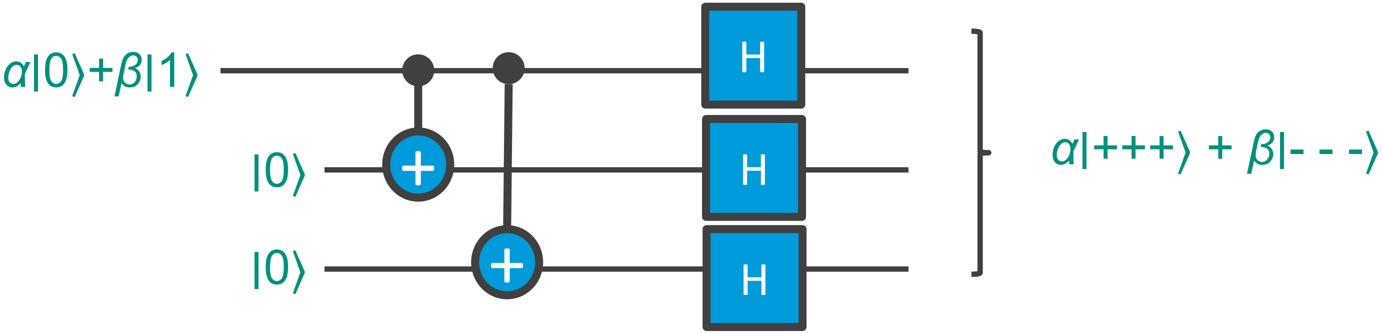
We can't use this to detect phase flip errors

CRPL

UNIVERSITY OF DELAWARE

Qubit repetition codes – **phase flip error**

The **3-bit repetition code** can be used to encode a qubit.

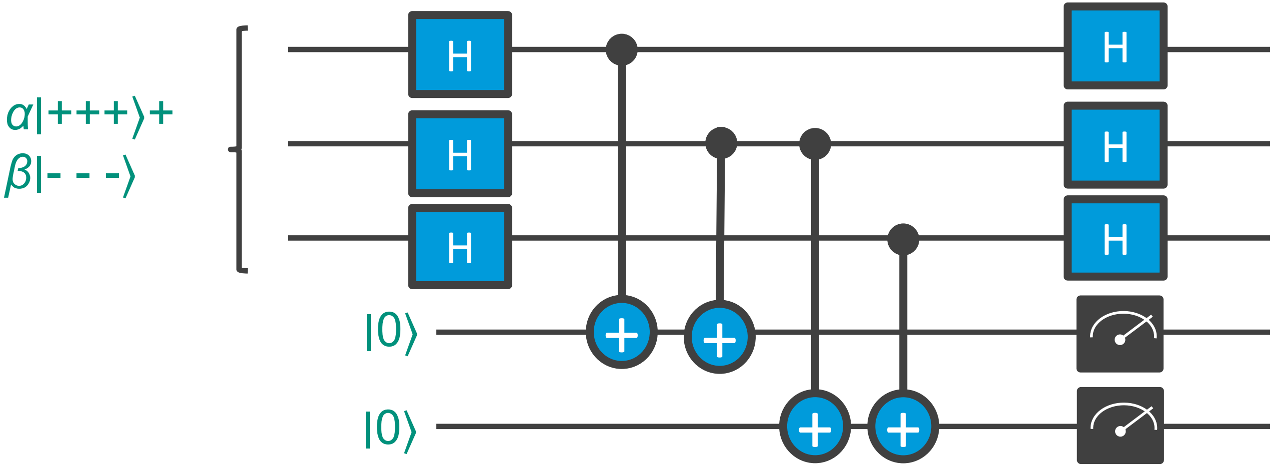


Phase flip errors or a z gate transforms (+ to −) or (− to +)

CRPL

UNIVERSITY OF DELAWARE

Qubit repetition codes – **phase** flip error

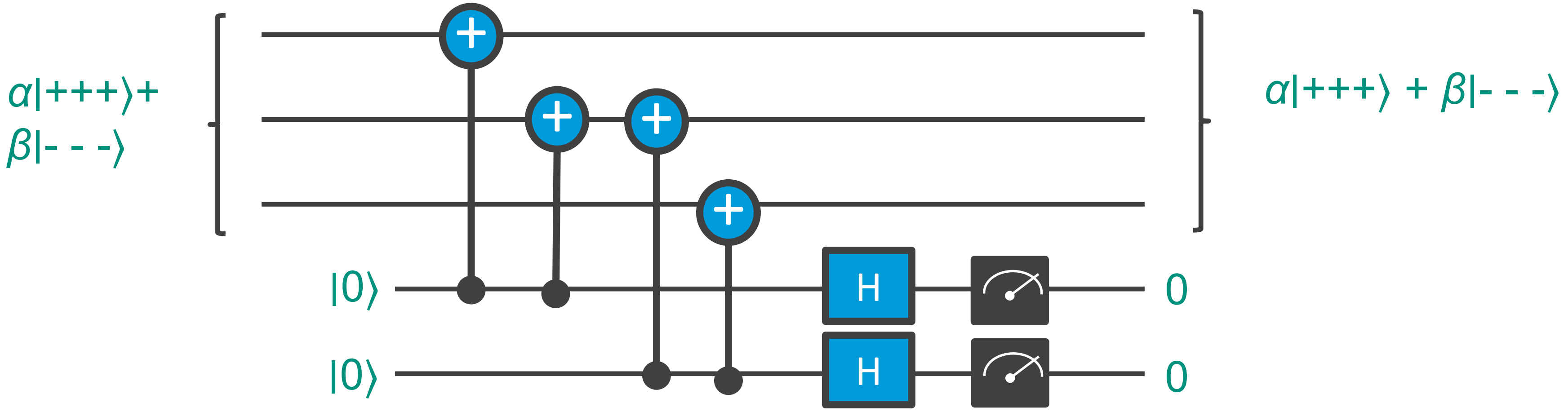


Add Hadamard gates at the beginning and end

CRPL

UNIVERSITY OF DELAWARE

Qubit repetition codes – **phase** flip error



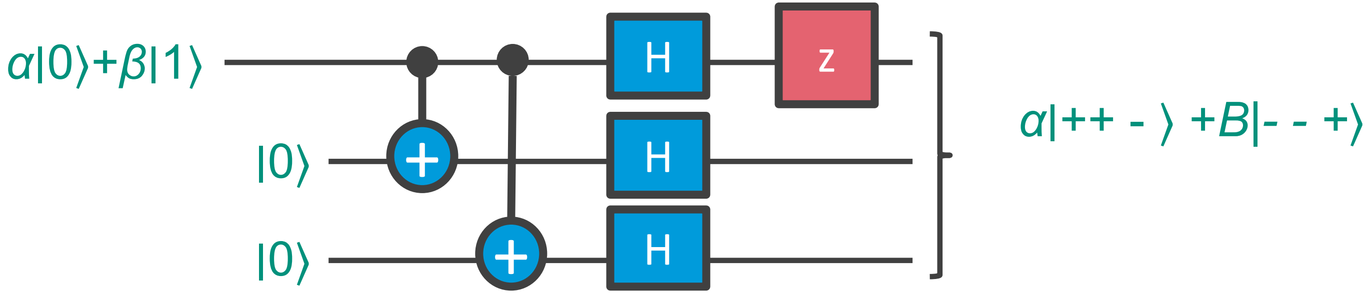
Without a phase flip error

CRPL

UNIVERSITY OF DELAWARE

Qubit repetition codes – **phase** flip error

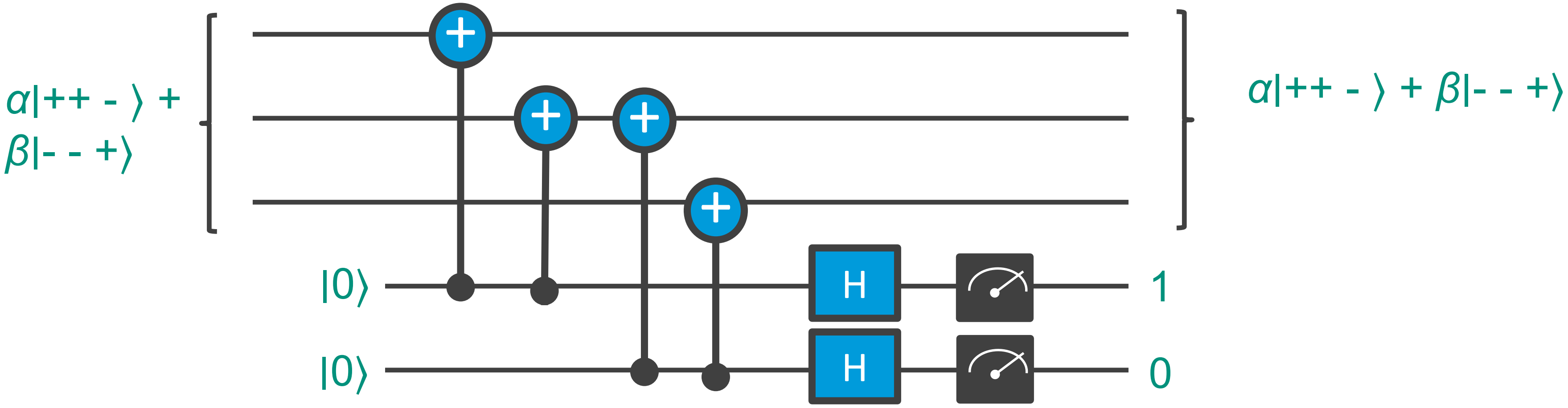
The **3-bit repetition code** can be used to encode a qubit.



CRPL

UNIVERSITY OF DELAWARE

Qubit repetition codes – **phase** flip error

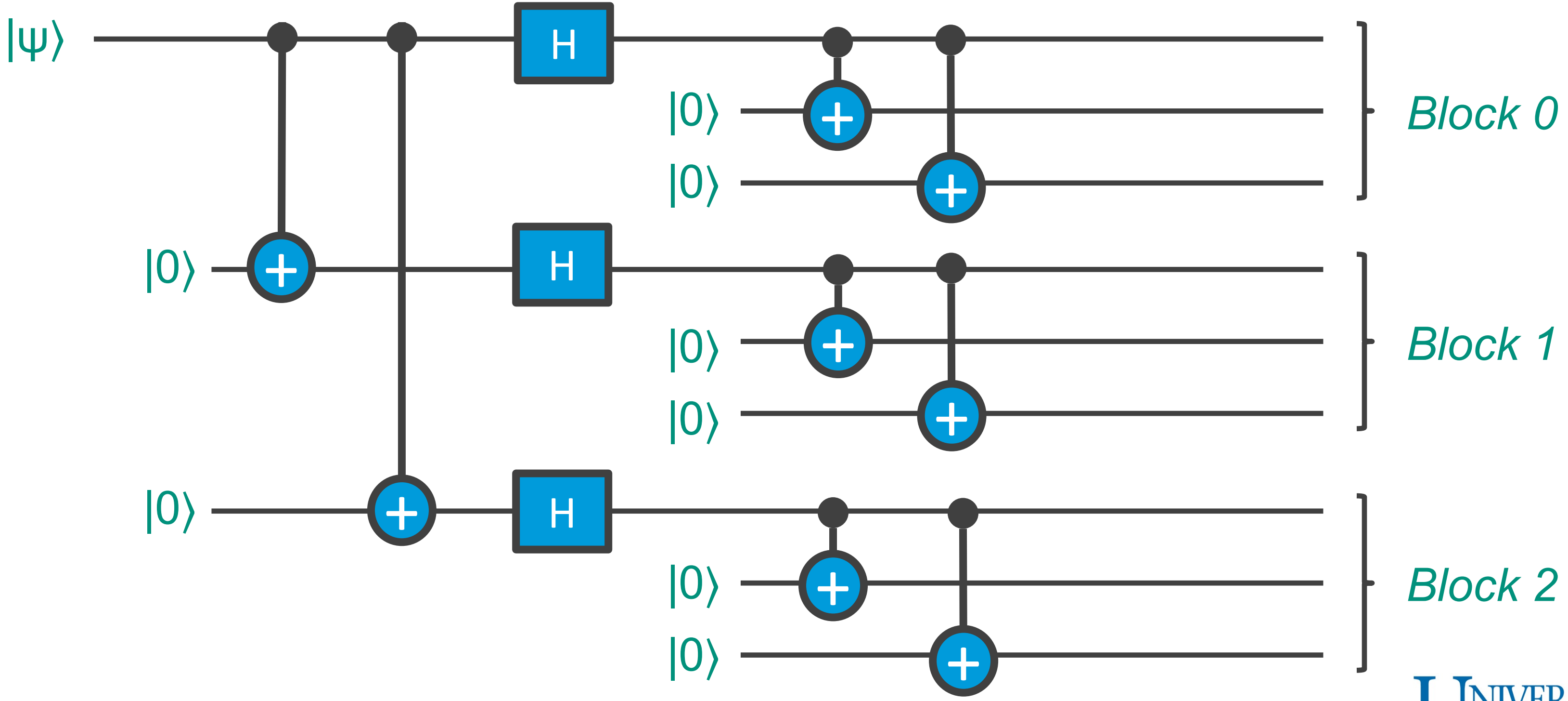


Allows us to detect a phase flip error

CRPL

UNIVERSITY OF DELAWARE

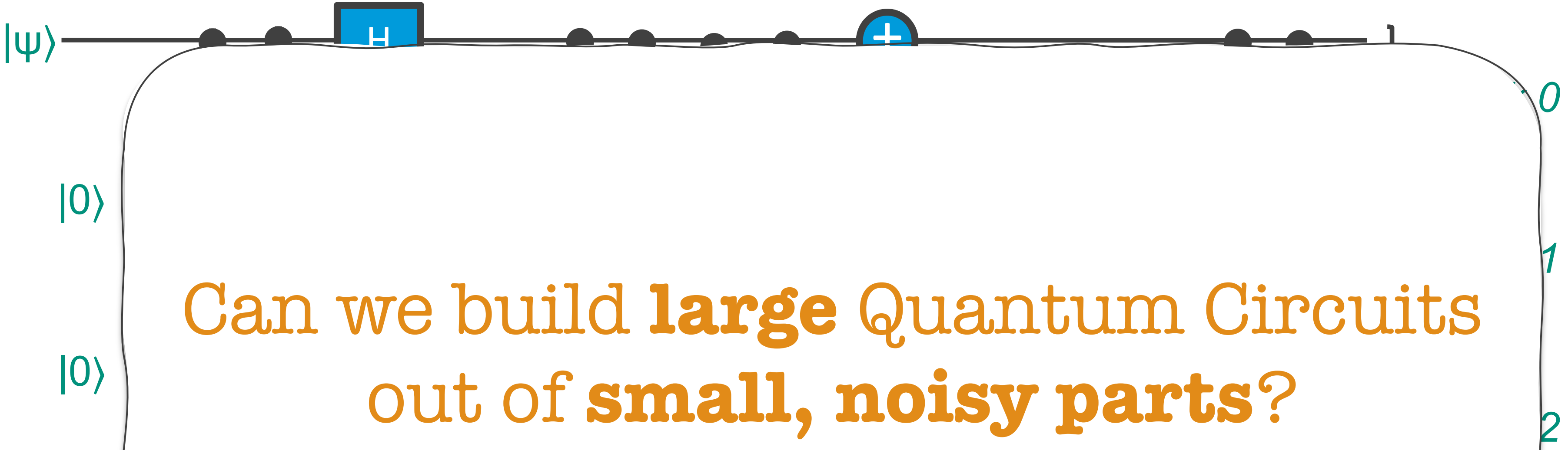
9-qubit Shor Code



CRPL

University of Delaware

9-qubit Shor Code



Can we build **large** Quantum Circuits out of **small, noisy parts**?

P. W. Shor (1995)

CRPL

University of Delaware

Timeline

1982
Richard P. Feynman

1995
Peter Shor's 9-qubit code

1996
Andrew Steane 7-qubit code (Logical Qubit)

1997
Alexei Kitaev Surface Code

1998
First experimental implementation of 3-bit code for phase error

First Experimental Quantum Error Correction

David G Cory, 1998

3-bit code for phase errors
Andrew Steane

$$|b_1 b_2 b_3\rangle \longrightarrow e^{i[(-1)^{b_1}\theta_1 + (-1)^{b_2}\theta_2 + (-1)^{b_3}\theta_3]}|b_1 b_2 b_3\rangle$$

liquid state Nuclear Magnetic Resonance

D. Cory, Et. Al.. (1998)

Fault-Tolerant Quantum Computing

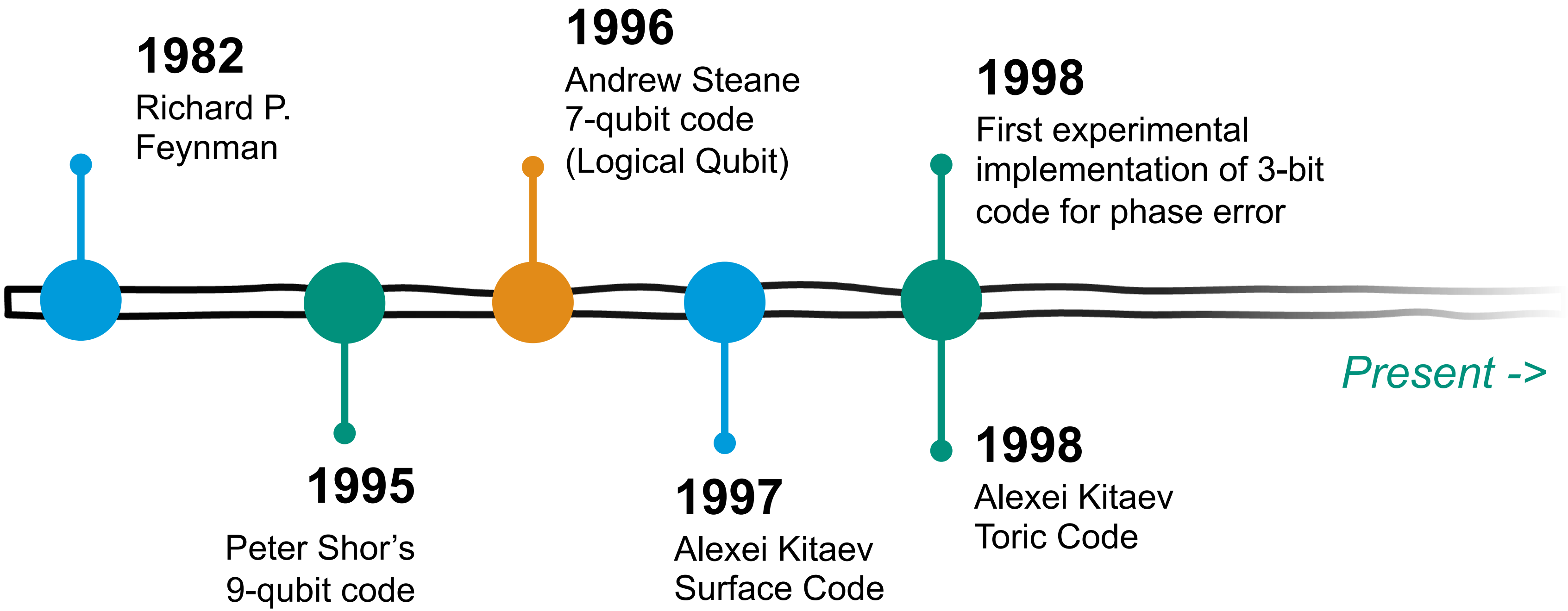
- **Fault-tolerance –** if only one component fails then the failure causes at most one error in each encoded block of qubits.

	2020	2023	2025	2027	2029
SYSTEMS:	SYSTEM MODEL H1	SYSTEM MODEL H2	QUANTINUUM HELIOS	QUANTINUUM SOL	QUANTINUUM APOLLO
PHYSICAL QUBITS:	20	56	96	192	1000's
PHYSICAL 2-QUBIT GATE ERROR:	1×10^{-3}	1×10^{-3}	$< 5 \times 10^{-4}$	$< 2 \times 10^{-4}$	1×10^{-4}
LOGICAL QUBITS:		> 12	~ 50	~ 100	100's
LOGICAL ERROR RATES:		1×10^{-3}	$< 10^{-4}$	$\sim 10^{-5}$	1×10^{-5} to $1 \times 10^{-10*}$

R. Delaney, Et. Al.. (2024)

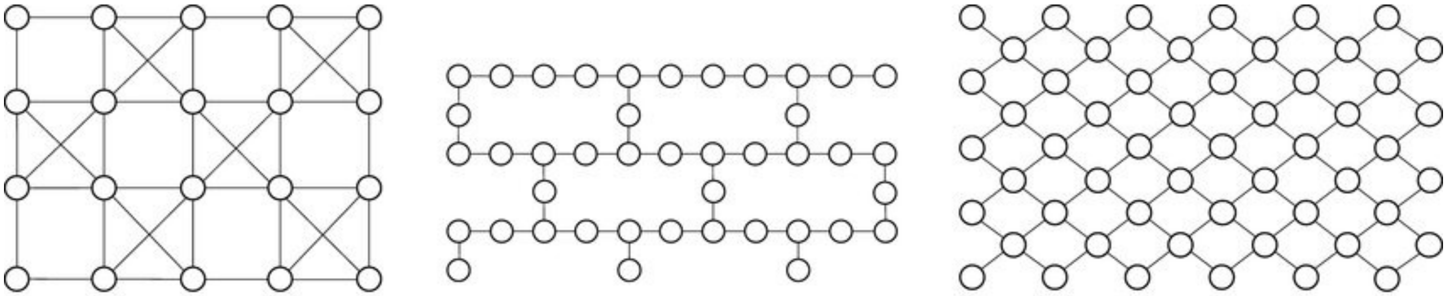
CRPL QUANTINUUM Microsoft UNIVERSITY OF DELAWARE

Timeline


1982
Richard P. Feynman
1995
Peter Shor’s 9-qubit code
1996
Andrew Steane 7-qubit code (Logical Qubit)
1997
Alexei Kitaev Surface Code
1998
First experimental implementation of 3-bit code for phase error
1998
Alexei Kitaev Toric Code
Present ->

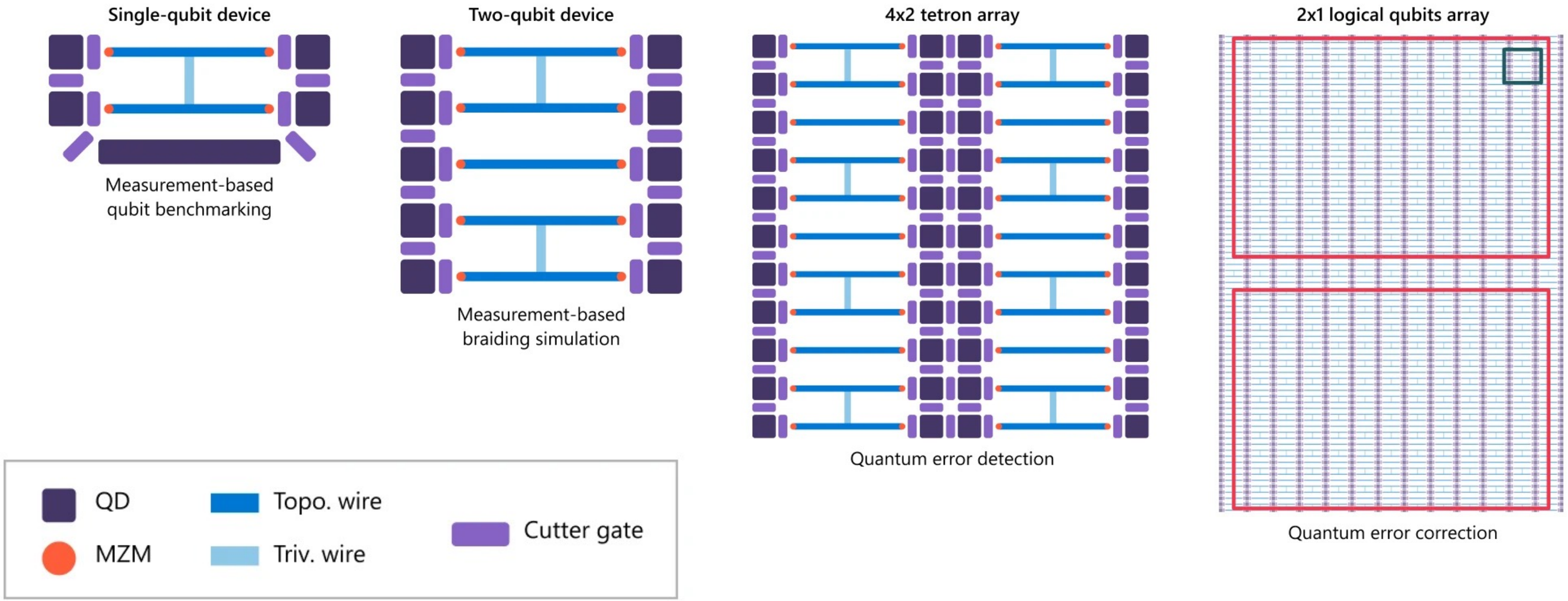
Topological Codes

Topological Code – spread out information over a 2D or 3D grid of qubits, removing small local errors



(a) Grid Topology (b) IBM's Heavy-hexagonal (c) Google's Sycamore

Topological Qubits



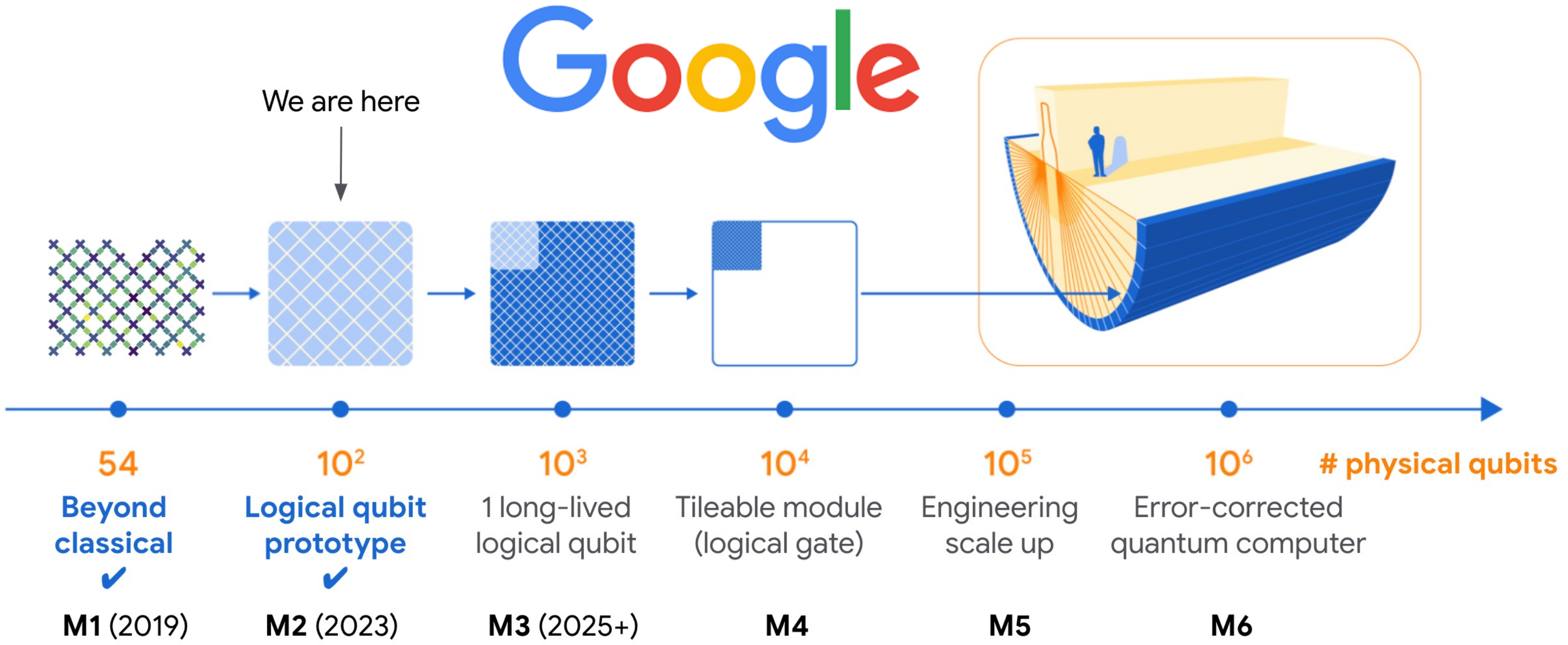
CRPL

C. Nayak. (2025)

Microsoft

University of Delaware

Google Quantum Error Correction



CRPL

UNIVERSITY OF DELAWARE

Refs

1. A. A. Saki, M. Alam, and S. Ghosh, “Study of Decoherence in Quantum Computers: A Circuit-Design Perspective,” *arXiv preprint arXiv:1904.04323*, 2019. [Online]. Available: https://arxiv.org/abs/1904.04323
2. Cory, D. G., Price, M. D., Maas, W., Knill, E., Laflamme, R., Zurek, W. H., Havel, T. F., & Somaroo, S. S. (1998). Experimental quantum error correction. *Physical Review Letters, 81*(10), 2152–2155. https://doi.org/10.1103/PhysRevLett.81.2152
3. Khalid, A. (2023). *A methods focused guide to quantum error correction and fault-tolerant quantum computation*.
4. Delaney, R. D., Sletten, L. R., Cich, M. J., Estey, B., Fabrikant, M. I., Hayes, D., Hoffman, I. M., Hostetter, J., Langer, C., Moses, S. A., Perry, A. R., Peterson, T. A., Schaffer, A., Volin, C., Vittorini, G., & Burton, W. C. (2024). Scalable multispecies ion transport in a grid-based surface-electrode trap. *Physical Review X, 14*(4), 041028. https://doi.org/10.1103/PhysRevX.14.041028